Scalar meson mass from renormalized One Boson Exchange Potentials[1]

A. Calle Cordón and E. Ruiz Arriola

Departamento de Física Atómica, Molecular y Nuclear,
Universidad de Granada,
E-18071 Granada, Spain.

Abstract. We determine the mass and strength of the scalar meson from *NN* scattering data by renormalizing the One Boson Exchange Potential. This procedure provides a great insensitivity to the unknown short distance interaction making the vector mesons marginally important allowing for SU(3) couplings in the $^1S_0$ channel. The scalar meson parameters are tightly constrained by low energy np. We discuss whether this scalar should be compared to the recent findings based on the Roy equations analysis of $\pi\pi$ scattering.

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INTRODUCTION

Half a century ago Johnson and Teller [1] suggested the need for a scalar-isoscalar meson with a mass ~ 500MeV to provide saturation and binding in nuclei. In a way this was the starting point for One-Boson-Exchange (OBE) Potentials where, in addition to the pion, all possible resonances would be included [2, 3, 4]. Despite their undeniable success describing NN scattering data, there has always been some arbitrariness on the scalar meson mass and coupling constant to the nucleon, partly stimulated by a lack of other sources of information, definitely helping the fits. The relation of the ubiquitous scalar meson in nuclear physics and NN forces in terms of correlated two pion exchange has been pointed out many times [2, 3] (see e.g. [5, 6, 7] for a discussion in a chiral context).

The quest for the existence of the 0^{++} resonance (commonly denoted by σ) has finally culminated with its inclusion in the PDG [8] as the $f_0(600)$ seen as a $\pi\pi$ resonance, where a spread of values ranging from $400-1200$MeV for the mass and a $600-1200$MeV for the width are displayed [9]. The uncertainties have recently been sharpened by a determination based on Roy equations and chiral symmetry [10] yielding the value $m_\sigma - i\Gamma_\sigma/2 = 441^{+16}_{-8} - i272^{+9}_{-12}$MeV; the lowest resonance in the hadronic spectrum. It is mandatory and perhaps possible to scrutinize its role in hadronic phenomenology all over. Here, we approach the problem from NN scattering in the $^1S_0$ channel from a renormalization viewpoint as applied to the OBE potential (without explicit inclusion of 2π exchange) and try to see the connection to $\pi\pi$ scattering.

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THE TRADITIONAL APPROACH TO OBE POTENTIALS

The field theoretical OBE model of the NN interaction [3] includes all mesons with masses below the nucleon mass, i.e., π, η, $\rho(770)$ and $\omega(782)$, in addition with a scalar-isoscalar boson. Dropping η and ρ because of their small couplings, the $^1S_0$ NN potential is

$$V(r) = -\frac{g_{\pi NN}^2 m_\pi^2}{16\pi M_N^2}\frac{e^{-m_\pi r}}{r} - \frac{g_{\sigma NN}^2}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g_{\omega NN}^2}{4\pi}\frac{e^{-m_\omega r}}{r} + \cdots \tag{1}$$

As is well known [2] any perturbative determination of a potential suffers from off-shell ambiguities (even in the Born approximation), particularly because of relativistic finite mass corrections which may be smoothly shifted between entirely energy dependent and local potentials or energy independent and nonlocal potentials. This trading between retardation and nonlocality may become sizable at short distances scales, where the interaction is unknown anyhow, and the particular choice is completely arbitrary. Our renormalization scheme will be such that, as suggested by Partovi and Lomon we ignore both retardation and nonlocality in the long distance limit [2], as well as the exponentially $\sim e^{-2M_N r}$ suppressed $N\bar{N}$ cut. Relativistic effects are only kept by renormalization of the couplings, but the effect is small [2].

In any case, one should bear in mind that NN scattering in the elastic region below pion production threshold involves CM momenta $p < p_{max} = 400$ MeV. Given the fact that $1/m_\omega = 0.25\text{fm} \ll 1/p_{max} = 0.5\text{fm}$ we expect heavier mesons to be irrelevant, and ω itself to be marginally important, even in s-waves, which are most sensitive to short distances. In order to illustrate this, we take $m_\pi = 138\text{MeV}$, $M_N = 939\text{MeV}$, $m_\omega = 783\text{MeV}$ and $g_{\pi NN} = 13.1$, which seem firmly established, and treat m_σ, $g_{\sigma NN}$ and $g_{\omega NN}$, as fitting parameters. As we show now, this vector meson irrelevance has not been fulfilled in the conventional approach to NN scattering, forcing too large $g_{\omega NN}$ couplings. Actually, in the standard approach the scattering phase-shift $\delta_0(p)$ is computed by solving the (s-wave) Schrödinger equation **r**-space

$$-u_p''(r) + M_N V(r)\, u_p(r) = p^2 u_p(r) \tag{2}$$

$$u_p(r) \rightarrow \frac{\sin(pr + \delta_0(p))}{\sin\delta_0(p)} \tag{3}$$

with a regular boundary condition at the origin $u_p(0) = 0$[3]. Moreover, for a short range potential such as the one in Eq. (1) one also has the Effective Range Expansion (ERE)

$$p\cot\delta_0(p) = -\frac{1}{\alpha_0} + \frac{1}{2} r_0\, p^2 + v_2\, p^4 + \cdots \tag{4}$$

where the *scattering length* α_0 and the *effective range* r_0 are defined by the asymptotic behavior of the zero energy wave function. In the usual approach [3, 4] everything is

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[2] This corresponds to $g_{\sigma NN}^2 \rightarrow g_{\sigma NN}^2/\sqrt{1 - m_\sigma^2/4M_N^2}$ and $g_{\omega NN}^2 \rightarrow g_{\omega NN}^2/\sqrt{1 - m_\omega^2/4M_N^2}$

[3] This boundary condition obviously implies a knowledge of the potential in the whole interaction region, and it is equivalent to solve the Lippmann-Schwinger equation in **p**-space.

obtained from the potential assumed to be valid for $0 \leq r < \infty$ [4]. In addition, due to the *unnaturally large* NN $^1S_0$ scattering length ($\alpha_0 \sim -23$fm), any change in the potential $V \to V + \Delta V$ has a dramatic effect on α_0, since one obtains

$$\Delta\alpha_0 = \alpha_0^2 M_N \int_0^\infty \Delta V(r) u_0(r)^2 \mathrm{d}r \tag{5}$$

and thus the potential parameters *must be fine tuned*, and in particular the short distance physics. A fit to the np data of Ref. [11] yields two possible but incompatible scenarios: $m_\sigma = 477.0(5)$MeV, $g_{\sigma NN} = 8.76(4)$, $g_{\omega NN} = 7.72(4)$ with $\chi^2/DOF = 0.85$ and $m_\sigma = 556.34(4)$MeV, $g_{\sigma NN} = 13.044(2)$, $g_{\omega NN} = 12.952(2)$ with $\chi^2/DOF = 0.52$. The small errors should be noted. The ambiguity in this solution is a typical inverse scattering one; note that despite the ω being repulsive, the total potential is not repulsive at short distances, and the corresponding couplings and scalar mass are determined to high accuracy but incompatible. This is just opposite to our expectations and we may regard these fits, despite their success in describing the data, as unnatural.

RENORMALIZATION OF THE OBE POTENTIAL

To overcome the unphysical short distance sensitivity we implement the renormalization viewpoint (see e.g. Ref. [12, 13]). In the simplest version one proceeds as follows

- For a given α_0 integrate in the zero energy wave function $u_0(r)$ down to the cut-off radius r_c. This is the renormalization condition.

$$-u_0''(r) + M_N V(r)\, u_0(r) \quad = \quad 0 \tag{6}$$

$$u_0(r) \quad \to \quad 1 - \frac{r}{\alpha_0} \tag{7}$$

- Impose self-adjointness to get the finite energy wave function $u_p(r_c)$,

$$u_p'(r_c)\, u_0(r_c) - u_0'(r_c) u_p(r_c) = 0 \tag{8}$$

- Integrate out the finite energy wave function $u_p(r)$, Eq. (2), to determine the phase shift $\delta_0(p)$ from Eq. (3).
- Remove the cut-off $r_c \to 0$ to ensure *model independence*.

This procedure allows us to compute $\delta_0(p)$ (and hence the next order's parameters r_0, v_2) from $V(r)$ and α_0 as independent information. Note that this is equivalent to consider, in addition to the regular solution, the irregular one[5]. A fit of the potential Eq. (1) to the np data of Ref. [11] using the renormalization method gets $m_\sigma = 490(18)$MeV, $g_{\sigma NN} = 8.8(6)$, $g_{\omega NN} = 0(9)$ with $\chi^2/DOF = 0.29$. Note that $g_{\omega NN}$ is, not only

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[4] In practice, strong form factors are included mimicking the finite nucleon size and reducing the short distance repulsion of the potential, but the regular boundary condition is always kept.

[5] In momentum space this can be shown to be equivalent to introduce one counterterm in the cut-off Lippmann-Schwinger equation, see Ref. [14] for a detailed discussion on this connection.

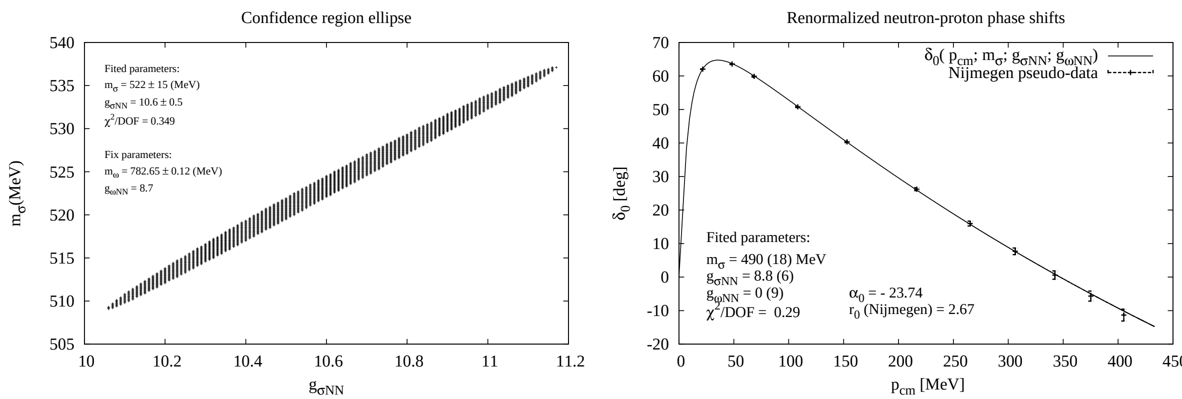

FIGURE 1. Left: $\Delta\chi^2 = 1$ confidence level ellipse in the $g_{\sigma NN} - m_\sigma$ plane for $g_{\omega NN} = 8.7$. Right: Renormalized OBE ${}^1S_0$ pn phase shifts (in degrees) as a function of CM momentum. Data from [11].

small but mostly irrelevant, so we consider this fit natural. A consequence of this is that we could take the SU(3) value $g_{\omega NN} = 3g_{\rho NN} - g_{\phi NN}$ which on the basis of the OZI rule, $g_{\phi NN} = 0$, Sakurai's universality $g_{\rho NN} = g_{\rho\pi\pi}/2$ and the KSFR relation $2g^2_{\rho\pi\pi} f^2_\pi = m^2_\rho$ yields $g_{\omega NN} \sim 8.7$ for which we get $m_\sigma = 522(10)\mathrm{MeV}$, $g_{\sigma NN} = 10.5(5)$ and $\chi^2/DOF = 0.3$ with a strong linear correlation (see Fig. 1).

WHAT SIGMA ?

Besides the numerical coincidence it is not obvious whether or not are we entitled to identify the NN-scalar with the $\pi\pi$-scalar because the $\pi\pi$-scalar has a large width, which suggests that this state decouples [6]. We suggest a large N_c motivated scenario where this identification might actually become compelling. The authors of [6] suggest that the potential due to iterated 2π scattering can be written for non-vanishing distances

$$V^C_{NN}(r) = -\frac{32\pi}{3m^4_\pi} \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot x} \left[\sigma_{\pi N}(-q^2)\right]^2 t_{00}(-q^2) \tag{9}$$

where $\sigma_{\pi N}(s)$ is the πN sigma term and $t_{00}(s) = (e^{2I\delta_{00}(s)} - 1)/(2i\sigma(s))$ the $\pi\pi$ scattering amplitude in the $I = J = 0$ channel as a function of the CM energy $\sqrt{s}$. In the large N_c limit, $t_{\pi\pi}(s) \sim 1/N_c$ while $\sigma_{\pi N}(s) \sim N_c$ yielding $V_{NN} \sim N_c$ as expected [15]. At the sigma pole

$$\frac{32\pi}{3m^4_\pi} [\sigma_{\pi N}(s)]^2 t^{II}_{\pi\pi}(s) \to \frac{g^2_{\sigma NN}}{s - (m_\sigma - i\Gamma_\sigma)^2} \to \frac{g^2_{\sigma NN}}{s - m^2_\sigma} \tag{10}$$

where in the second step we have taken the large N_c limit. This yields $g_{\sigma\pi\pi} \sim 1/\sqrt{N_c}$, provided $m_\sigma \sim N^0_c$ and $\Gamma_\sigma \sim 1/N_c$, a point disputed in Ref. [16] where the IAM method

[6] A simple modification such as $V_\sigma(r) \to V_\sigma(r)\cos(\Gamma_\sigma r/2)$ provides an inadmissible mid-range repulsion.

is applied to $\pi\pi$ scattering. If we use instead the Bethe-Salpeter method to lowest order [17], we get a once subtracted dispersion relation, with an arbitrary constant

$$t_{00}^{-1}(s) - t_{00}^{-1}(4m_\pi^2) = v_{00}^{-1}(s) - v_{00}^{-1}(4m_\pi^2) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds'\,\sigma(s')\left[\frac{1}{s-s'} - \frac{1}{s-4m_\pi^2}\right] \quad (11)$$

where $v_{00}(s) = (m_\pi^2 - 2s)/(32\pi^2 f_\pi^2)$ is the tree level amplitude and $\sigma(s) = (1-4m_\pi^2/s)^{\frac{1}{2}}$ the two-pion phase space. The difference between $t_{00}(4m_\pi^2)$ and $v_{00}(4m_\pi^2)$ is higher order in the chiral expansion but both scale as $1/N_c$. We fix the accurately determined scattering length $-t_{00}(4m^2) = a_{00}m = 0.220(2)$ [18, 19]. For $f_\pi = 92.3$MeV, and $m = 139.6$MeV we get the pole at $m_\sigma - i\Gamma_\sigma/2 = 467 - i192$MeV although $\delta_{00} = 50^o$ at $E_{\pi\pi} = 500$MeV overshoots the Roy analysis value $\sim 35(5)^0$ [19] mainly because higher order chiral corrections [17] and possibly subthreshold KK effects [20], have been omitted. Scaling according to large N_c counting $a_{00} \to \sqrt{3/N_c}a_{00}$ and $f_\pi \to \sqrt{N_c/3}f_\pi$ the unitarity integral in Eq. (11) can be neglected and the pole satisfies

$$-(a_{00}m_\pi)^{-1} = v_{00}^{-1}(m_\sigma^2) - v_{00}^{-1}(4m_\pi^2) \quad (12)$$

The limit is smooth, and while $\Gamma_\sigma \to 0$ we get $m_\sigma \to 506.8$MeV, closer to the NN-scalar. On view of this agreement it is tempting to think that perhaps the σ proposed by Johnson and Teller in 1955 might correspond to the σ determined by Caprini, Colangelo and Leutwyler in 2006 *in the large N_c* limit. It remains to be seen if higher order chiral and $1/N_c$ corrections both for NN as well as for $\pi\pi$ support this view.

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REFERENCES

1. M. H. Johnson, and E. Teller, *Phys. Rev.* **98**, 783–787 (1955).
2. M. H. Partovi, and E. L. Lomon, *Phys. Rev.* **D2**, 1999–2032 (1970).
3. R. Machleidt, K. Holinde, and C. Elster, *Phys. Rept.* **149**, 1–89 (1987).
4. R. Machleidt, *Phys. Rev.* **C63**, 024001 (2001).
5. N. Kaiser, S. Gerstendorfer, and W. Weise, *Nucl. Phys.* **A637**, 395–420 (1998).
6. E. Oset, H. Toki, M. Mizobe, and T. T. Takahashi, *Prog. Theor. Phys.* **103**, 351–365 (2000).
7. J. F. Donoghue, *Phys. Lett.* **B643**, 165–170 (2006).
8. W. M. Yao, et al., *J. Phys.* **G33**, 1–1232 (2006).
9. E. van Beveren, F. Kleefeld, G. Rupp, and M. D. Scadron, *Mod. Phys. Lett.* **A17**, 1673 (2002).
10. I. Caprini, G. Colangelo, and H. Leutwyler, *Phys. Rev. Lett.* **96**, 132001 (2006).
11. V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, *Phys. Rev.* **C49**, 2950–2962 (1994).
12. M. Pavon Valderrama, and E. R. Arriola, *Phys. Rev.* **C74**, 054001 (2006).
13. M. Pavon Valderrama, and E. R. Arriola (2007) *0705.2952 (nucl-th)* .
14. D. R. Entem, E. Ruiz Arriola, M. Pavon Valderrama, and R. Machleidt (2007) *0709.2770 (nucl-th)*.
15. D. B. Kaplan, and A. V. Manohar, *Phys. Rev.* **C56**, 76–83 (1997).
16. J. R. Pelaez, *Phys. Rev. Lett.* **92**, 102001 (2004).
17. J. Nieves, and E. Ruiz Arriola, *Nucl. Phys.* **A679**, 57–117 (2000).
18. J. Nieves, and E. Ruiz Arriola, *Eur. Phys. J.* **A8**, 377–384 (2000).
19. G. Colangelo, J. Gasser, and H. Leutwyler, *Nucl. Phys.* **B603**, 125–179 (2001).
20. J. A. Oller, E. Oset, and J. R. Pelaez, *Phys. Rev. Lett.* **80**, 3452–3455 (1998).